

Analysis of Algorithm

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(Week 14) Lectures 27 & 28

Objectives: Learning objectives of these lectures are

- Graph Colouring
- Example
- Application

Text Book & Resources:

1. Introduction to Algorithms by Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein, The MIT Press; 3rd Edition (2009). ISBN-10: 0262033844
2. Introduction to the Design and Analysis of Algorithms by Anany Levitin, Addison Wesley; 2nd Edition (2006). ISBN-10: 0321358287
3. Algorithms in C++ by Robert Sedgewick (1999). ASIN: B006UR4BJS
4. Algorithms in Java by Robert Sedgewick, Addison-Wesley Professional; 3rd Edition (2002). ISBN-10: 0201361205

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In the last lecture (**Week#13**), we discussed “Graph and MST” and problems in hashing and how to resolve these problems. In this week, we will discuss “Graph Coloring” and their Examples and also its Application.

Graph Coloring:

Graph coloring refers to the problem of coloring vertices of a graph in such a way that no two adjacent vertices have the same color.

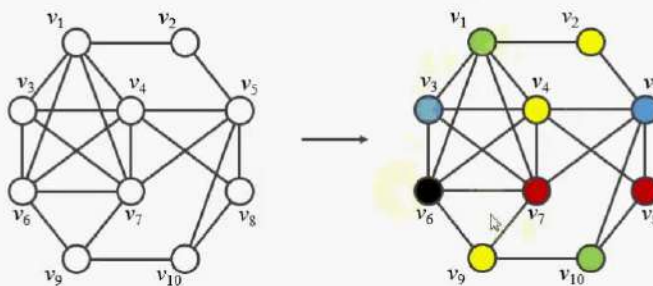
This is also called the vertex coloring problem.

If coloring is done using at most m colors, it is called m -coloring.

Example:

Now colour the vertices of the graph so that:

- No adjacent vertices are allocated the same colour
- The number of colours used is minimised



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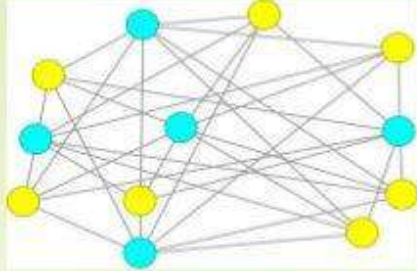
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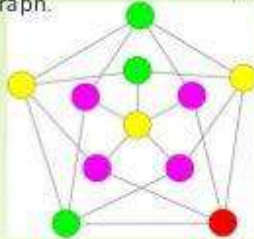
Example:

❖ A graph is 2-colorable iff it is bipartite



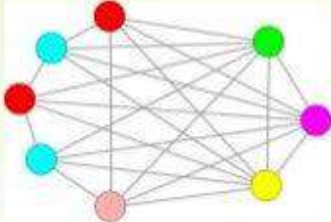
❖ Mycielski's Construction

- It Can be used to make graphs with arbitrarily large chromatic numbers, that do not contain K_3 as a sub graph.



❖ $\omega(G)$ – size of largest clique in G
 $\chi(G) \geq \omega(G)$

- Clique of size n requires n colors
- $\chi(G)=7, \omega(G)=5$.



Algorithm:

Algorithm: Greedy Graph Coloring

Input:

$G = (V, E)$

An undirected simple graph

Output:

A vertex coloring for graph G

```
1:  begin
2:      Let  $\{v_1, v_2, \dots, v_n\}$  be a sequence of the vertices in  $V$ .
3:      for  $v := v_1$  to  $v_n$ 
4:          begin
5:              Assign vertex  $v$  the smallest possible color such that
              no conflict exists between  $v$  and its colored neighbors.
6:          end
7:  end
```

Time Complexity: $O(m^n)$

Space Complexity: $O(n)$

For the recursion stack in backtracking. Additionally,
 $O(n+e)$ is required for storing the graph, where e is the number of edges.

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Applications:

- Scheduling

Graph coloring can be used to assign tasks or events to different time slots or resources. For example, you can use graph coloring to schedule exams or create a timetable.

- Frequency allocation

Graph coloring can be used to assign frequencies to different devices so that no two adjacent devices use the same frequency. For example, you can use graph coloring to assign radio frequencies to stations.

- Map coloring

Graph coloring can be used to assign colors to regions on a map so that adjacent regions have different colors.

- Wireless communication

Graph coloring can be used to assign frequencies to different devices so that no two adjacent devices use the same frequency.

