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(Week12) Lecture23-24

Objectives: **Learning objectives of this lecture are**

- **FSA's With Outputs**
  - **MELAY MACHINES**
    - **Examples**
  - **MOORE MACHINES**
    - **Examples**
  - **MEALY TO MOORE AND MOORE TO MEALY CONVERSION Rules**
    - **Conversion Examples**

Text Book &Resources: Introduction to Computer Theory–2nd Edition-(I. O. Cohen)

Video Resources link

https://www.youtube.com/playlist?list=PLOIUSe71PzPG_YxnHL_MosWvn6DwztzNJ

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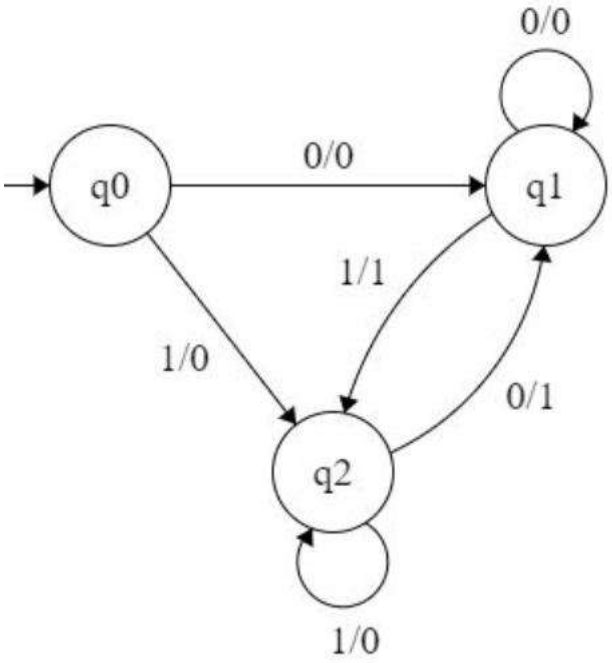
Finite State Automata with output

Finite State Automata have always with some final states that accept words of a particular language. Now we will discuss FSA's with outputs called Mealy Machines and Moore Machines. In this FSA, we have a separate output alphabet set which may or may not equal to input alphabets sets.

Mealy Machines

Mealy machines are finite state machines with output value and its output depends on present state and current input symbol. It can be defined as (Q, q0, ∑, O, δ, λ) where:

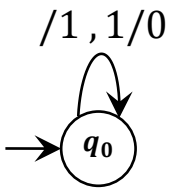
- Q is finite set of states.
- q0 is the initial state.
- ∑ is the input alphabet.
- O is the output alphabet.
- δ is transition function which maps Q×∑→Q.
- 'λ' is the output function which maps Q×∑→O.



In the mealy machine shown in Figure above the output is represented with each input symbol for each state separated by /. The length of output for a mealy machine is equal to the length of input.

- Input:11
- Transition: δ(q0,1)=>δ(q2,1)=>q2
- Output:00 (q0 to q2 transition has Output 0 and q2 to q2 transition also has Output 0)

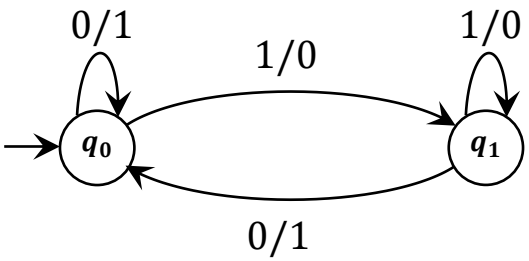
Example1: Provide a Mealy Machine gives complement of a given input binary stream.



state	q_0	q_0	q_0	q_0	q_0	q_0	q_0	q_0
i/p	-	0	0	1	1	1	0	1
o/p	-	1	1	0	0	0	1	0

Above table shows the output of the complement mealy machine. Here input was 0011101 having output 1100010 for the input.

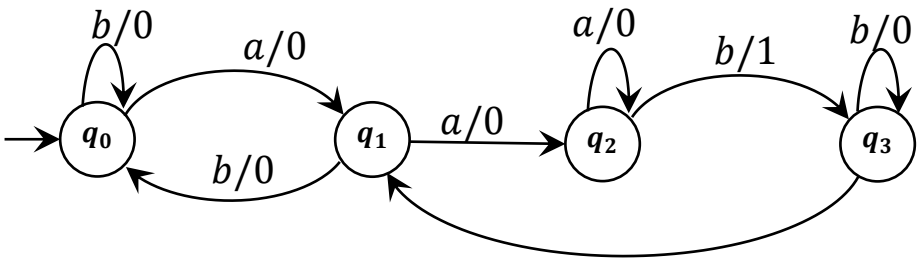
Example2: Provide a Mealy Machine gives complement of a given input binary stream.(Another solution).



state	q_0	q_0	q_0	q_1	q_1	q_1	q_0	q_1
i/p	-	0	0	1	1	1	0	1
o/p	-	1	1	0	0	0	1	0

Above table shows theoutputofthecomplementmealymachine.Hereinputwas0011101havingoutput1100010fortheinput

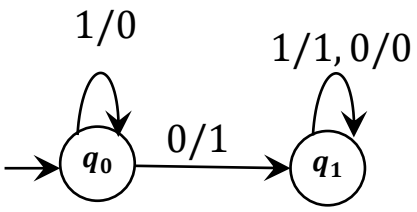
Example3: Design a Mealy machine that beep on 'aab'



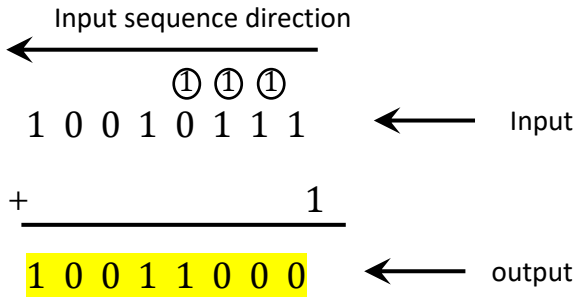
state	q_0	q_1	q_0	q_1	q_0	q_1	q_2	q_3
i/p	-	a	b	a	b	a	A	b
o/p	-	0	0	0	0	0	0	1

Initially, we are on state q_0 having no input and no output. Now on giving input *a*, the next state will be q_1 and having output 0. And finally, on input string *ababaab*, output string will be 0000001.

Example4: Make a Mealy machine that add 1 in input.

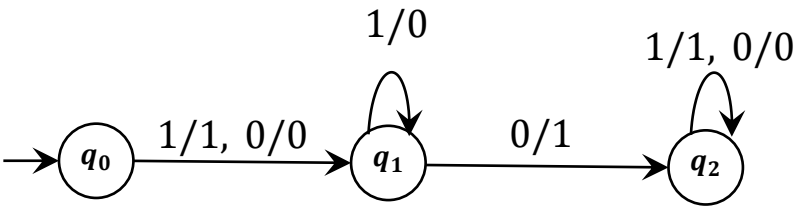


In above figure, initial state will be operated on the input from right to left. For example, for the input sequence 10010111 the operation will be



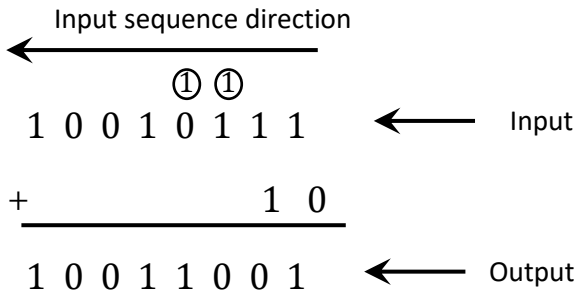
state	q_0	q_0	q_0	q_0	q_1	q_1	q_1	q_1	q_1
i/p	-	1	1	1	0	1	0	0	1
o/p	-	0	0	0	1	1	0	0	1

Example5: Make a Mealy machine that increment 2 in input.



In above figure, initial state will be operated on the input from right to left.

For example, for the input sequence 10010111 the operation will be

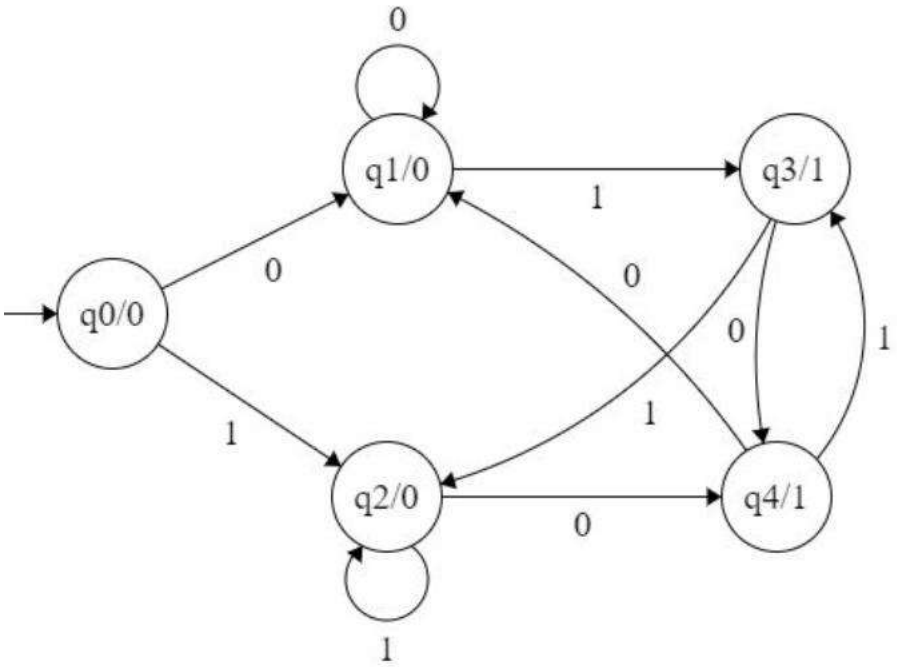


state	q_0	q_1	q_1	q_1	q_2	q_2	q_2	q_2	q_2
i/p	-	1	1	1	0	1	0	0	1
o/p	-	1	0	0	1	1	0	0	1

Moore Machines

Moore machines are finite state machines with output value and its output depends only on present state. It can be defined as (Q, q0, ∑, O, δ, λ) where:

- Q is finite set of states.
- q0 is the initial state.
- ∑ is the input alphabet.
- O is the output alphabet.
- δ is transition function which maps Q×∑→Q.
- λ is the output function which maps Q→O.

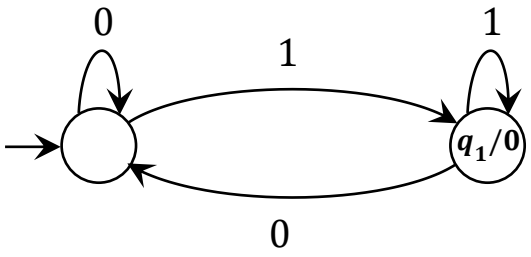


In the moore machine shown in Figure, the output is represented with each input state separated by / . The length of output for a moore machine is greater than input by 1.

- Input:11

- Transition: δ(q0,1)=>δ(q2,1)=>q2
- Output:000 (0 for q0, 0 for q2 and again 0 for q2)

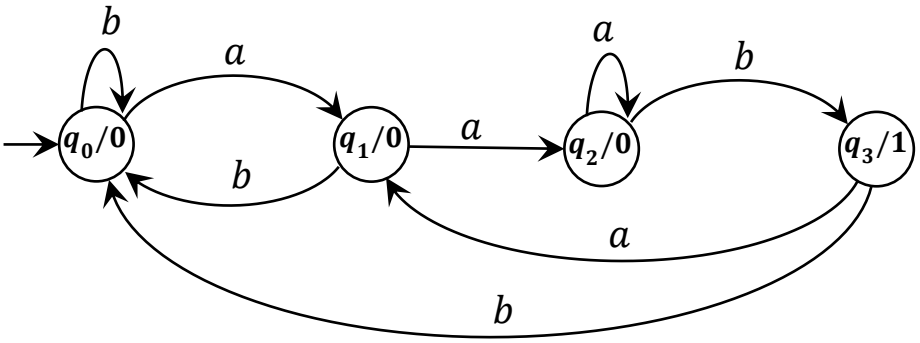
Example6: Provide a Moore Machine gives complement of a given input binary stream.



state	q_0	q_0	q_0	q_1	q_1	q_1	q_0	q_1
i/p	-	0	0	1	1	1	0	1
o/p	1	1	1	0	0	0	1	0

Above table shows the output of the complement moore machine. Here input was 0011101 having output 11100010 for the input. Highlighted bit is always ignore in Moore machines. It was produced on first entry of start state when no input was given. So it will always be ignored.

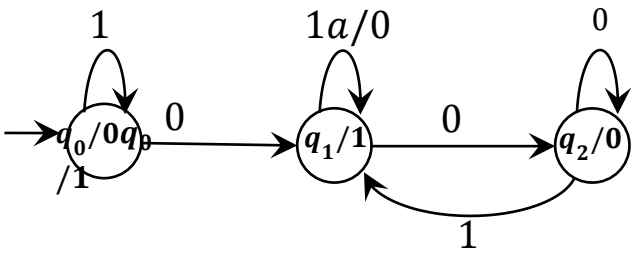
Example7: Design a Moore machine that beep on 'aab'



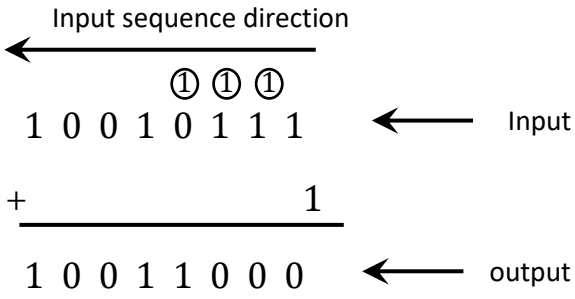
state	q_0	q_1	q_0	q_1	q_0	q_1	q_2	q_3
i/p	-	a	b	a	b	a	a	b
o/p	0	0	0	0	0	0	0	1

Initially, we are on state q_0 having no input and output as 0. Now on giving input a, then state will be q_1 and having output 0. And finally, on input string *ababaab*, output string will be 00000001.

Example8: Make a Moore machine that add 1 in input.



In above figure, initial state will be operated on the input from right to left. For example, for the input sequence 10010111 the operation will be



state	q_0	q_0	q_0	q_0	q_1	q_1	q_2	q_2	q_1
i/p	-	1	1	1	0	1	0	0	1
o/p	0	0	0	0	1	1	0	0	1

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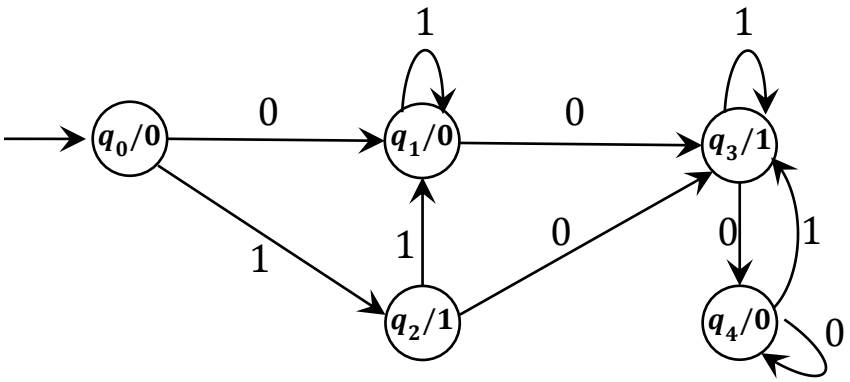
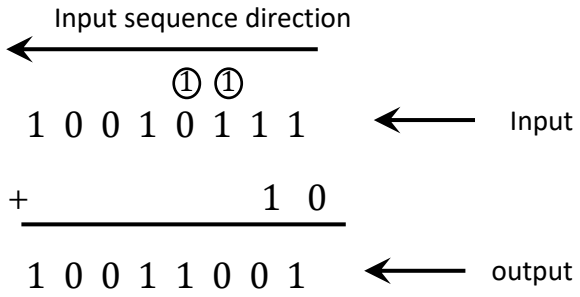
Example9: Make a Moore machine that increment 2 in input.



In above figure, initial state will be operated on the input from right to left.

For example, for the input sequence 10010111 the operation will be



state	q_0	q_2	q_1	q_1	q_3	q_3	q_4	q_4	q_3
i/p	-	1	1	1	0	1	0	0	1
o/p	0	1	0	0	1	1	0	0	1

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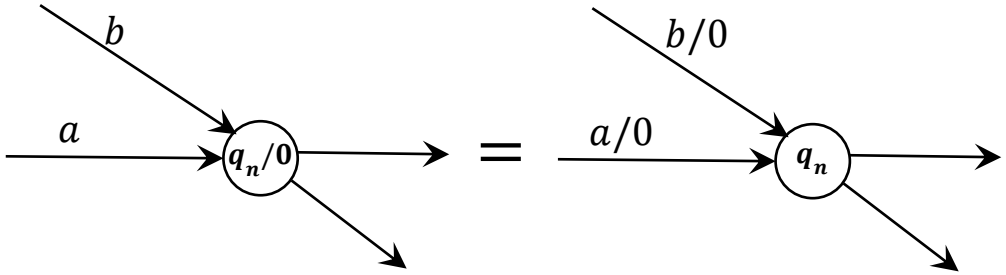
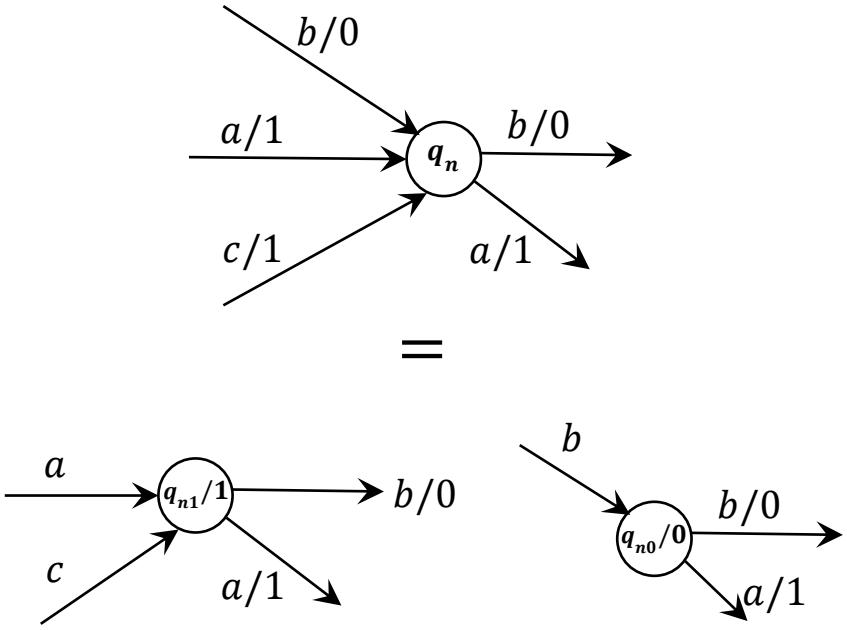
Mealy Machine to Moore Machine Conversion

We can follow some rules to convert from Mealy to Moore

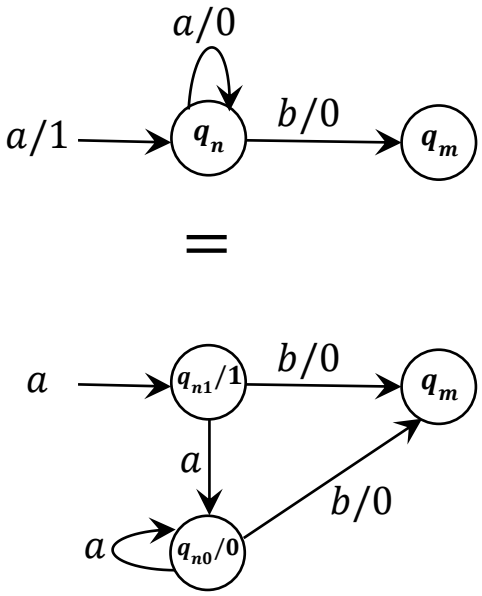
Rule 1



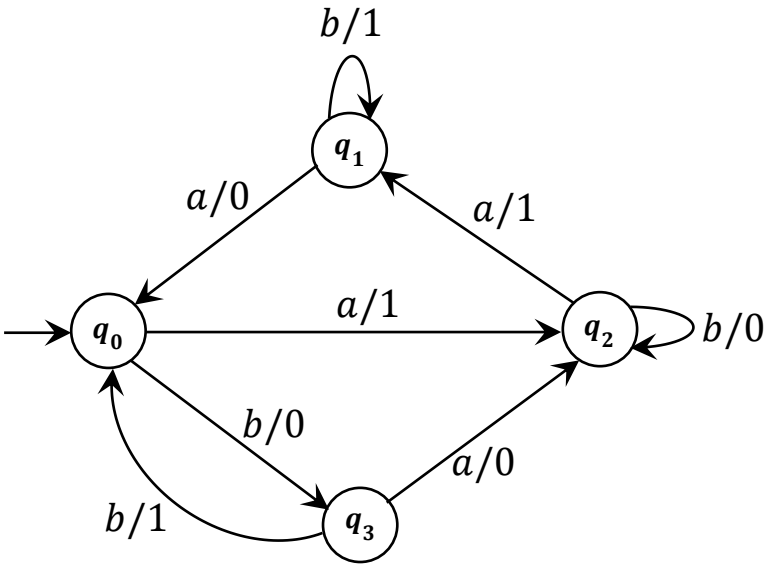
Rule 2

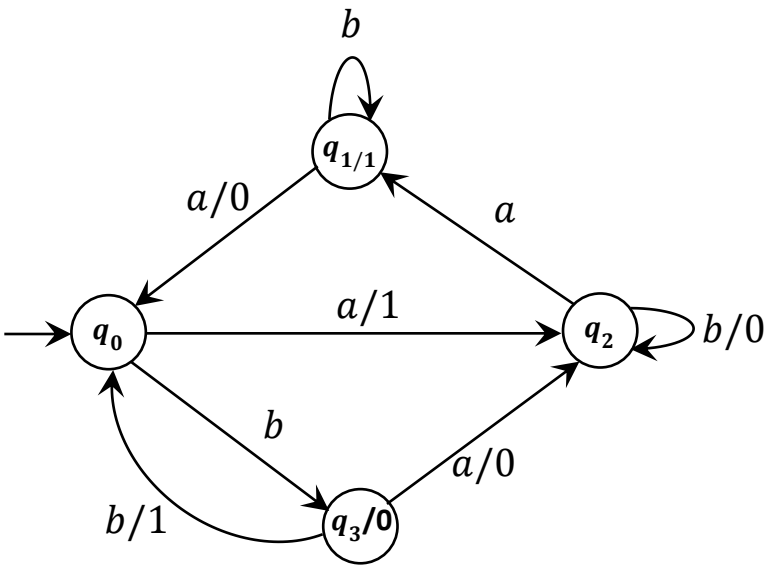


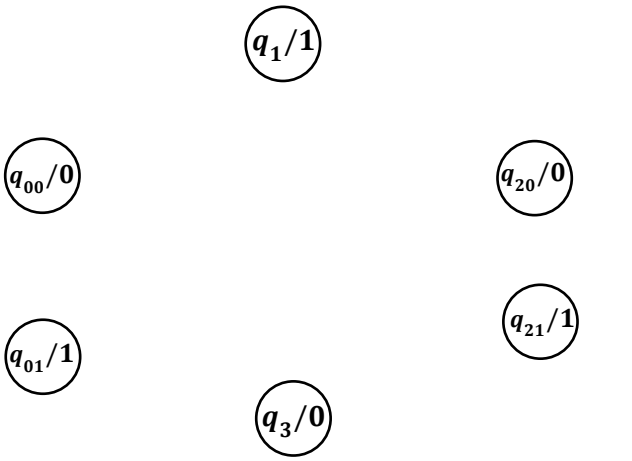
Rule 3

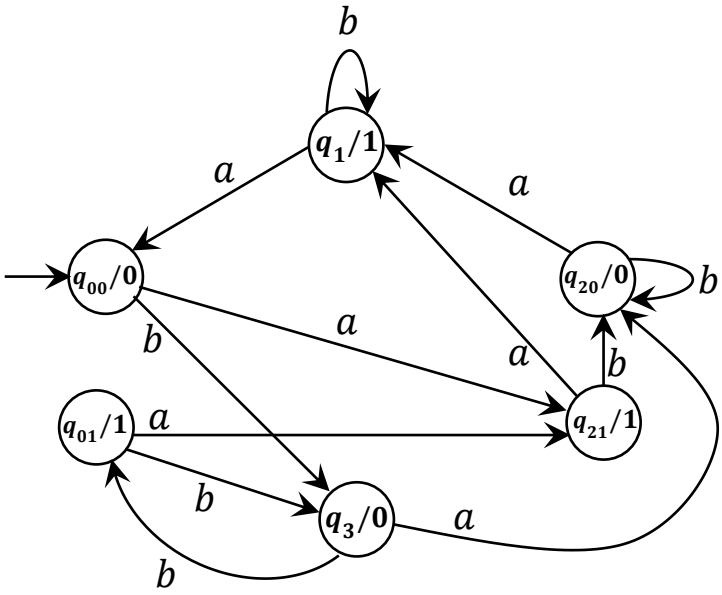


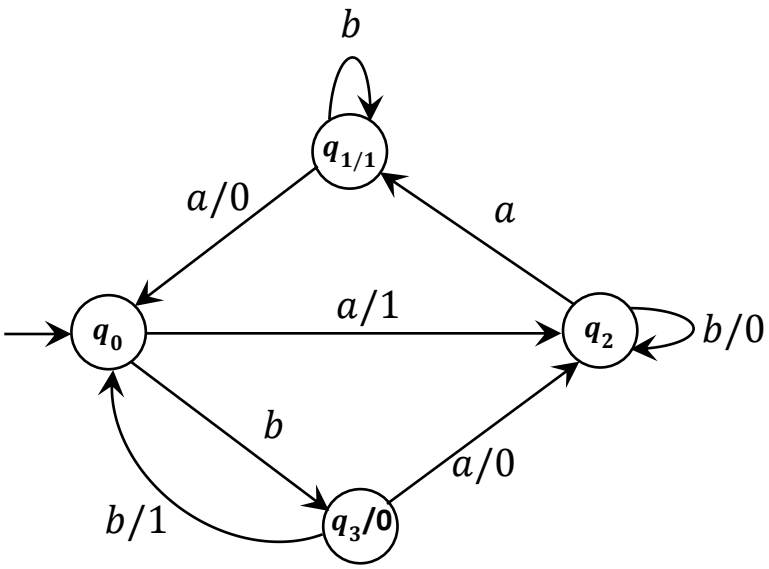
Example 10: Convert the following Mealy machine to Moore machine?




b
$q_{1/1}$
a/0
a
a/1
q_0
q_2
b/0
b
a/0
b/1
q_3/0


$q_1/1$
$q_{00}/0$
$q_{20}/0$
$q_{21}/1$
$q_{01}/1$
$q_3/0$


b
$q_1/1$
a
a
$q_{00}/0$
$q_{20}/0$
b
b
a
a
b
$q_{01}/1$
a
$q_{21}/1$
b
$q_3/0$
a
b


b
$q_{1/1}$
a/0
a
a/1
q_0
q_2
b/0
b
a/0
b/1
q_3/0