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(Week13)Lecture25-26

Objectives: **Learningobjectivesofthislectureare**

- **Context Free Languages**
- **Context Free Grammar**
- **Syntax Trees**
- **Ambiguity**
- **CFG For Regular Expression**
- **CFG For FSA**

TextBook&Resources:IntroductiontoComputerTheory–2nd Edition-(I.O.Cohen)

Video Recources link

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Context Free Language

Definition:

- The language generated by a CFG is the set of all strings of terminals that can be produced from the start symbol S using the productions as substitutions.
- A language generated by a CFG is called a context-free language (CFL).

Notes:

- The language generated by a CFG is also called the language defined by the CFG, or the language derived from the CFG, or the language produced by the CFG.
- We insist that non terminals be designated by capital letters, whereas terminals are designated by lowercase letters and special symbols.

Definition of terms

Terminal: A word that cannot be replaced by anything is called terminal.

Non-Terminal: A word that must be replaced by other things is called non terminal.

Derivation: The sequence of applications of the rules that produces the finished string of terminals from the starting symbol is called a derivation or a generation of the word.

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Context Free Grammar

A context-free grammar (CFG) is a collection of three things:

- An alphabet $\sum$ of letters called terminals from which we are going to make strings that will be the words of a language.
- A set of symbols called non terminals, one of which is the symbol S, standing for "start here".
- A finite set of productions of the form:

  One non terminal → finite string of terminals and/or non terminals.

Example 1: The Language of a^* ={null, a,aa,aaa,aaaa,......}

Prod1 $S \rightarrow aS$

Prod2 $S \rightarrow \Lambda$

If we want to generate the $aaaaaa$ then the following steps used

$$S \Rightarrow aS$$

$\Rightarrow aaS$

$\Rightarrow aaaS$

$\Rightarrow aaaaS$

$\Rightarrow aaaaaS$

$\Rightarrow aaaaaaS$

$\Rightarrow aaaaaa\Lambda$

$\Rightarrow aaaaaa$

Example 2: The Language of Palindrome $\Sigma = \{a, b\}$.

Prod1	$S \rightarrow aSa$
Prod2	$S \rightarrow bSb$
Prod3	$S \rightarrow a$
Prod4	$S \rightarrow b$
Prod5	$S \rightarrow \Lambda$

Now by passing the word *abaaba*

$$S \Rightarrow aSa$$

$$\Rightarrow abSba$$

$$\Rightarrow abaSaba$$

$$\Rightarrow aba\Lambda aba$$

$$\Rightarrow abaaba$$

Example 3: The Language of Palindrome $\Sigma = \{a, b\}$ can also be shown like this

$$S \rightarrow aSa \mid bSb \mid a \mid b \mid \Lambda$$

Example 4: The Language of Palindrome $\Sigma = \{a, b\}$ can also be shown like this

$$S \rightarrow ASA \mid BSB \mid A \mid B \mid \Lambda$$

$$A \rightarrow a$$

$$B \rightarrow b$$

Now by passing the word *abaaba*

$$S \Rightarrow ASA$$

$$\Rightarrow ABSBA$$

$$\Rightarrow ABASABA$$

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$\Rightarrow aba\Lambda aba$

$\Rightarrow abaaba$

Example 5: The Language $(a+b)^*$ defined over $\Sigma = \{a, b\}$.

$$S \rightarrow aS \mid bS \mid \Lambda$$

The word $abaa$ can be represented by using above CFG

$S \Rightarrow aS$

$\Rightarrow abS$

$\Rightarrow abaS$

$\Rightarrow abaaS$

$\Rightarrow abaa\Lambda$

$\Rightarrow abaa$

Example 6: The Language contain substring aa defined over $\Sigma = \{a, b\}$

$S \rightarrow XaaX$ (a+b)*aa(a+b)*

$X \rightarrow aX \mid bX \mid \Lambda$

Now by passing the word $abbaab$

$S \Rightarrow XaaX$

$\Rightarrow aXaaX$

$\Rightarrow abXaaX$

$\Rightarrow abbXaaX$

$\Rightarrow abb\Lambda aaX$

$\Rightarrow abbaaX$

$\Rightarrow abbaabX$

$\Rightarrow abbaab\Lambda$

$\Rightarrow abbaab$

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Syntax Trees

Example7: The Language start with a and end on b defined over $\Sigma = \{a, b, c\}$.

$$S \rightarrow aXb$$

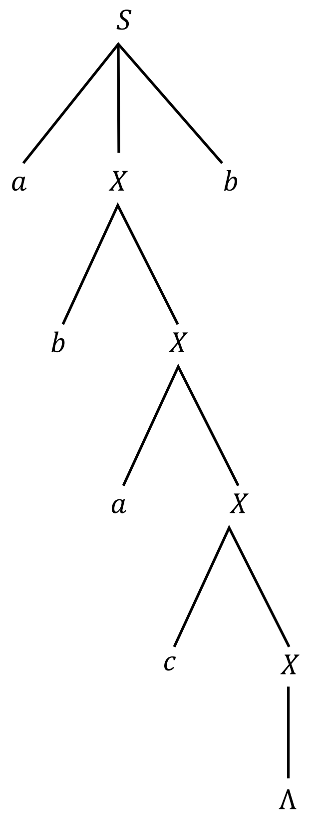
a (a+b+c)*b

$$X \rightarrow aX \mid bX \mid cX \mid \Lambda$$

Now by passing the word $abacb$

$$S \Rightarrow aXb$$
$$\Rightarrow abXb$$
$$\Rightarrow abaXb$$
$$\Rightarrow abacXb$$
$$\Rightarrow abac\Lambda b$$
$$\Rightarrow abacb$$

Another form to represent word generations is Syntax Trees



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Example 8: The Language $\{a^n b^* a^n\}$ for $n = 1,2, \ldots$defined over $\Sigma = \{a, b\}$.

$$S \to aSa \mid aXa$$

$$X \to bX \mid \Lambda$$

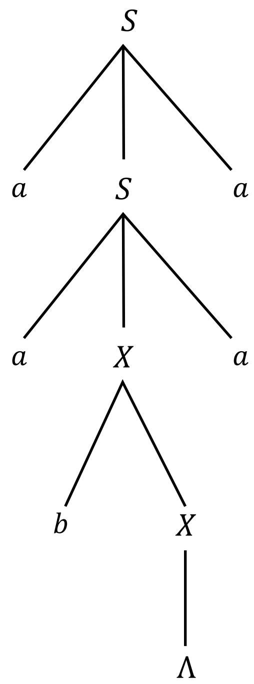
S->aSa | X
X-> bX | null

Now by passing a word $aabaa$

$$S \Rightarrow aSa$$
$$\Rightarrow aaXaa$$
$$\Rightarrow aabXaa$$
$$\Rightarrow aab\Lambda aa$$
$$\Rightarrow aabaa$$

S=>aSa
=>aaSaa
=>aaXaa
=>aabXaa
=>aabaa

Another form to represent word generations is Syntax Trees.



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Example 9: The Language $\{a^n b^{2n}\}$ for $n =$ 0, 1,2, … defined over $\Sigma = \{a, b\}$.

S-> aSbb | null

$$S \to aSB \mid \Lambda$$

$$B \to bb$$

The word $aabbbb$ can be passed

$$S \Rightarrow aSB$$

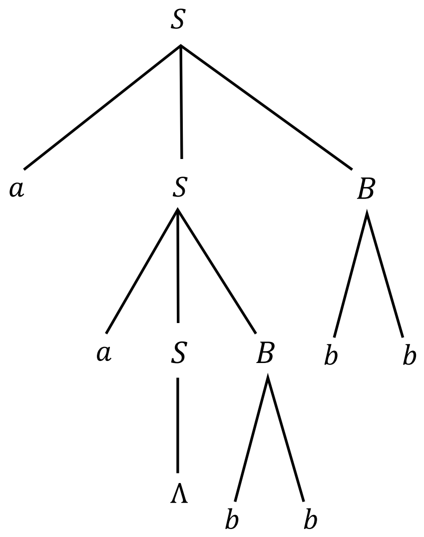
Left Most Derivation (LMD)

$$\Rightarrow aaSBB$$

$$\Rightarrow aa\Lambda BB$$

$$\Rightarrow aabbB$$

$$\Rightarrow aabbbb$$



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Example 10: The Language $\{a^{2n}b^n\}$ for $n = 0,1,2,3,\dots$ defined over $\Sigma = \{a, b\}$.

S->aaSb | null

S=>aaSb
=>aaaaSbb
=>aaaabb

$S \rightarrow aA \mid \Lambda$

$A \rightarrow aSB \mid aB$

$B \rightarrow b$

S->ASb | null
A->aa

The word $aaaabb$ can be passed

$S \Rightarrow aA$

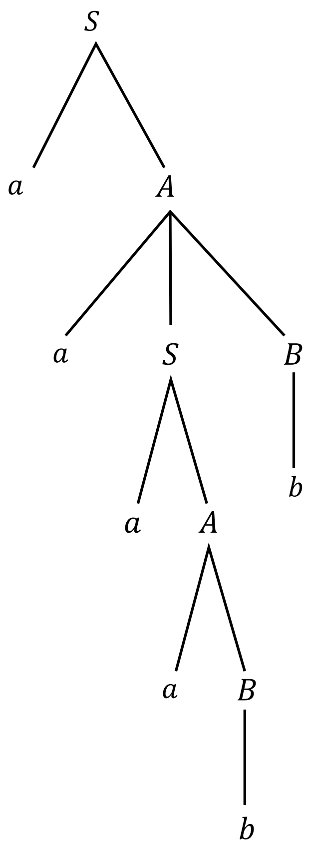
$\Rightarrow aaSB$

$\Rightarrow aaaAB$

$\Rightarrow aaaaBB$

$\Rightarrow aaaabb$

S=>ASb
=>AASbb
=>aaASbb
=>aaaaSbb
=>aaaabb



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<u>Example 11:</u> The Language $\{a^n b^m a^{n+m}\}$ for $n, m = 0,1,2,3, \ldots, \Sigma = \{a, b\}$.

$$S \rightarrow aSa \mid L$$

$$L \rightarrow bLa \mid \Lambda$$

Now by passing the word $aabaaa$

$$S \Rightarrow aSa$$

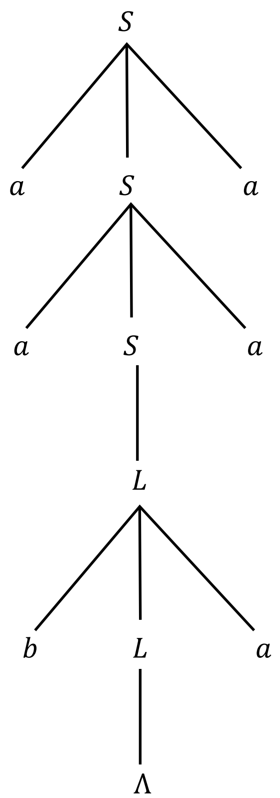
$$\Rightarrow aaSaa$$

$$\Rightarrow aaLaa$$

$$\Rightarrow aabLaaa$$

$$\Rightarrow aab\Lambda aaa$$

$$\Rightarrow aabaaa$$



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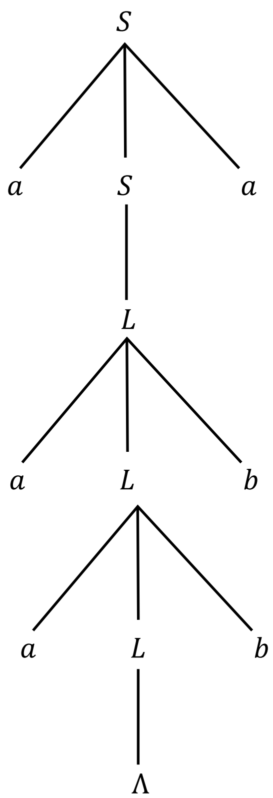
Example 12: The Language $\{a^n b^m a^{n-m}\}$ for $n, m = 0,1,2,3, \ldots$, $\Sigma = \{a, b\}$.

$$S \rightarrow aSa \mid L$$

$$L \rightarrow aLb \mid \Lambda$$

Now by passing the word $aaabba$

$$S \Rightarrow aSa$$
$$\Rightarrow aLa$$
$$\Rightarrow aaLba$$
$$\Rightarrow aaaLbba$$
$$\Rightarrow aaa\Lambda bba$$
$$\Rightarrow aaabba$$



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Ambiguity

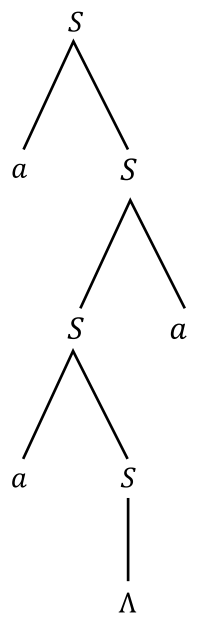
A CFG is called ambiguous if for at least one word in the language that it generates there are two possible derivations of the word that correspond to different syntax trees.

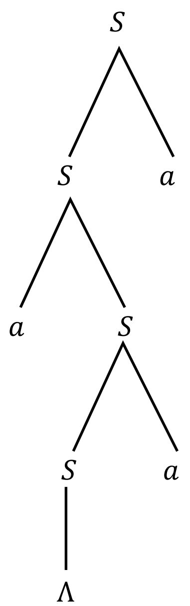
Example 13: The CFG

$$S \to aS \mid Sa \mid \Lambda$$

for the Language of a^* defined over $\Sigma = \{a\}$ is ambiguous because for same word aaa, different derivations and syntax trees exist as shown below

$S \Rightarrow aS$	$S \Rightarrow Sa$
$\Rightarrow aSa$	$\Rightarrow aSa$
$\Rightarrow aaSa$	$\Rightarrow aSaa$
$\Rightarrow aa\Lambda a$	$\Rightarrow a\Lambda aa$
$\Rightarrow aaa$	$\Rightarrow aaa$





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RE to CFG Conversion

Each regular expression can be converted into CFG's

Example 14: Write CFG for the given RE

$$\text{RE} = a(a+b)^*(aa+bb)$$

Solution: The Regular expression given is for the language starts with a and end on double letter defined over $\Sigma = \{a, b\}$.

$$S \to aX$$

$$X \to aX \mid bX \mid aa \mid bb$$

Here

$$S \to aX$$ Starts with a

$$X \to \boxed{aX \mid bX} \mid \boxed{aa \mid bb}$$

$(a+b)^*$ $(aa+bb)$

Example 15: Write CFG for the given RE

$$\text{RE} = (ab+ba)(a+b)^*(ab+ba)$$

Solution: The Regular expression given is for the language cannot starts andend with double letter defined over $\Sigma = \{a, b\}$.

S->abXY|baXY
X->aX|bX|null
Y->ab|ba

abaabbab

$$S \to abX \mid baX$$

$$X \to aX \mid bX \mid ab \mid ba$$

Here

S->abX
->abaX
->abaaX
->abaabX
->abaabbX
->abaabbab

$$S \to abX \mid baX$$ Cannot Start with double letter

$$X \to \boxed{aX \mid bX} \mid \boxed{ab \mid ba}$$

$(a+b)^*$ Cannot end with double

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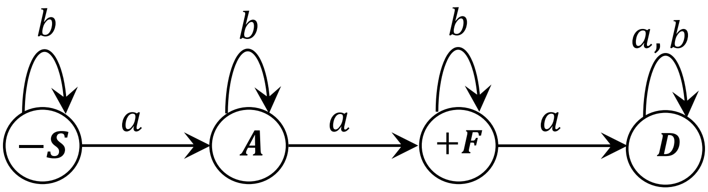
FSA to CFG Conversion

Example 15: Write CFG for the FSA given below



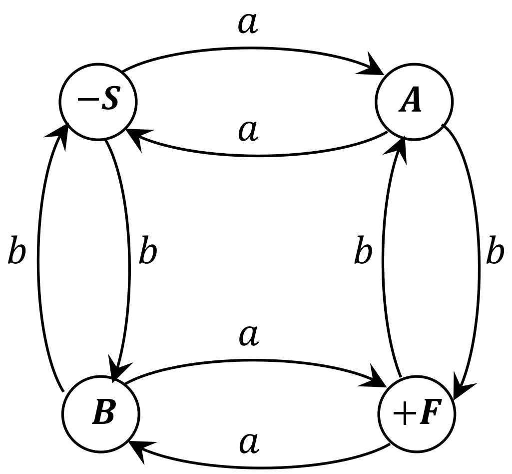
Solution:

$S \rightarrow bS \mid aA$

$A \rightarrow bA \mid aF$

$F \rightarrow bF \mid \Lambda$

Example 16: Write CFG for the FSA given below



Solution:

$S \rightarrow aA \mid bB$

$A \rightarrow aS \mid bF$

$B \rightarrow aF \mid bS$

$F \rightarrow aB \mid bA \mid \Lambda$

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Assignment

Q#1: Write CFG for the Language $\{a^n b^* a^n\}$ for $n = 0,1,2,3, \dots$, $\Sigma = \{a, b\}$.

Q#2: Write CFG for the Language $\{a^n b^{2n}\}$ for $n = 3,4,5, \dots$, $\Sigma = \{a, b\}$.

Q#3: Write CFG for the Language $\{a^{4n} b^n\}$ for $n = 1,2,3, \dots$, $\Sigma = \{a, b\}$.

Q#4: Write CFG for the Language $\{a^{n+2} b^n\}$ for $n = 2,3, \dots$, $\Sigma = \{a, b\}$.