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(Week14)Lecture27-28

Objectives: **Learningobjectivesofthislectureare**

- **Simplification of CFG's**
  - **Reduction of CFG**
  - **Removal of Unit Productions**
  - **Removal of Null Productions**

TextBook&Resources:IntroductiontoComputerTheory–2nd Edition-(I.O.Cohen)

Video Recources link

Spmlification of CFG

Definition:

In CFG, sometimes all the production rules and symbols are not needed for the derivation of strings. Beside this, there may also be some NULL Productions and UNIT Productions. Elimination of these productions and symbols is called simplification of CFG. Simplification consists of the following steps.

- Reduction of CFG.
- Removal of Unit Productions.
- Removal of NULL Productions.

Reduction of CFG

In simplification of CFG, there are three main steps involved. First step is Reduction of CFG. Reduction of CFG further done in two phases. In first phase, drive an equivalent grammar G' from the CFG G such that each variable drives some terminal string. In second phase, drive an equivalent grammar G'' from the CFG G' such that each symbol appears in a sentential form. Now the steps involved in both phases will be as follows

Phase 1:

Derivation of an equivalent grammar G' from the CFG G such that each variable drives some terminal string.Derivation Procedure involves the following steps as given below

Step 1: Include all symbols W_1, that derives some terminal and initialize $i = 1$

Step 2: Include symbols W_{i+1}, that derives W_i.

Step 3: Increment i and repeat Step 2, until $W_{i+1} = W_i$

Step 4: Include all production rules that have W_i in it.

Phase 2:

Derivation of an equivalent grammar G'' from the CFG G' coming from Phase 1,such that each symbol appears in a sentential form. Derivation Procedure involves the following steps as given below

Step 1: Include the Start symbol in y_1 and initialize $i = 1$.

Step 2: Include all symbols y_{i+1}, that can be derived from y_i and include all production rules that have been applied.

Step 3: Increment i and repeat Step 2, until $y_{i+1} = y_i$

Example 1

Find a reduced grammar equivalent to the grammar G, having production rules as

$$S \rightarrow AC \mid B$$

$$A \rightarrow a$$

$$C \rightarrow c \mid BC$$

$$E \rightarrow aA \mid e$$

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Solution:

Phase 1

$$T = \{a, c, e\}$$

$$W_1 = \{A, C, E\}$$

$$W_2 = \{A, C, E, S\}$$

$$W_3 = \{A, C, E, S\}$$

$$G' = \{(A, C, E, S), \{a, c, e\}, P, (S)\}$$

P: $S \to AC$

$A \to a$

$C \to c$

$E \to aA \mid e$

Phase 2

$$y_1 = \{S\}$$

$$y_2 = \{S, A, C\}$$

$$y_3 = \{S, A, C, a, c\}$$

$$y_4 = \{S, A, C, a, c\}$$

$$G'' = \{(A, C, S), \{a, c\}, P, (S)\}$$

P: $S \to AC$

$A \to a$

$C \to c$

Example 2

Find a reduced grammar equivalent to the grammar G, having production rules as

$$S \to AB$$

$$A \to a$$

$$B \to b \mid D$$

$$E \to d$$

Solution:

Phase 1

$$\boldsymbol{T = \{a, b, d\}}$$

$$\boldsymbol{W_1 = \{A, B, E\}}$$

$$\boldsymbol{W_2 = \{A, B, E, S\}}$$

$$\boldsymbol{W_3 = \{A, B, E, S\}}$$

$$\boldsymbol{G' = \{(A, B, E, S), \{a, d, e\}, P, (S)\}}$$

$$\boldsymbol{P}: \quad \boldsymbol{S \to AB}$$

$$\boldsymbol{A \to a}$$

$$\boldsymbol{B \to b}$$

$$\boldsymbol{E \to d}$$

Phase 2

$$\boldsymbol{y_1 = \{S\}}$$

$$\boldsymbol{y_2 = \{S, A, B\}}$$

$$\boldsymbol{y_3 = \{S, A, B, a, b\}}$$

$$\boldsymbol{y_4} = \{\boldsymbol{S}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{a}, \boldsymbol{b}\}$$

$$\boldsymbol{G}'' = \{(\boldsymbol{S}, \boldsymbol{A}, \boldsymbol{B}), \{\boldsymbol{a}, \boldsymbol{b}\}, \boldsymbol{P}, (\boldsymbol{S})\}$$

$$\boldsymbol{P}: \quad \boldsymbol{S} \to \boldsymbol{AB}$$

$$\boldsymbol{A} \to \boldsymbol{a}$$

$$\boldsymbol{B} \to \boldsymbol{b}$$

Removal of Unit Productions

Any Production Rule of the form $A \to B$ where $A, B \in$ Non-Terminals is called Unit Production.For removal of Unit Productions, steps given below will be followed.

Step 1: To remove $A \to B$, add production $A \to x$ to the grammar rule whenever $B \to x$ occur in the grammar. where

$$x \in Terminal, \qquad and\ x\ can\ be\ Null$$

Step 2: Delete $A \to B$ from the grammar.

Step 3:Repeat from Step 1 until all Unit Productions are removed.

Example 3

Remove Unit Productions from the grammar whose production rules are given by

$$S \rightarrow XY$$

$$X \rightarrow a$$

$$Y \rightarrow Z \mid b$$

$$Z \rightarrow M$$

$$M \rightarrow N$$

$$N \rightarrow a$$

Solution:

Since $N \rightarrow a$, we add $M \rightarrow a$

$$S \rightarrow XY$$

$$X \rightarrow a$$

$$Y \rightarrow Z \mid b$$

$$Z \rightarrow M$$

$$M \rightarrow a$$

$$N \rightarrow a$$

Since $M \rightarrow a$, we add $Z \rightarrow a$

$$S \rightarrow XY$$

$$X \rightarrow a$$

$$Y \rightarrow Z \mid b$$

$$Z \rightarrow a$$

$$M \rightarrow a$$

$$N \to a$$

Since $Z \to a$, we add $Y \to a$

$$S \to XY$$

$$X \to a$$

$$Y \to a \mid b$$

$$Z \to a$$

$$M \to a$$

$$N \to a$$

Since $X \to a$, we add $S \to aY$

$$S \to aY$$

$$X \to a$$

$$Y \to a \mid b$$

$$Z \to a$$

$$M \to a$$

$$N \to a$$

Since $Y \to a \mid b$, we add $S \to aa \mid ab$

$$S \to aa \mid ab$$

$$X \to a$$

$$Y \to a \mid b$$

$$Z \to a$$

$$M \to a$$

$$N \to a$$

Now remove the unreachable symbols, so production Rules will become

$$S \to aa \mid ab$$

Example 4

Remove Unit Productions from the grammar whose production rules are given by

$$S \to XY$$

$$X \to Y$$

$$Y \to Z \mid b$$

$$Z \to bM$$

$$M \to Nb$$

$$N \to a$$

Solution:

Since $N \to a$, we add $M \to ab$

$$S \to XY$$

$$X \to Y$$

$$Y \to Z \mid b$$

$$Z \to bM$$

$$M \to ab$$

$$N \to a$$

Since $M \to ab$, we add $Z \to bab$

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$$S \rightarrow XY$$

$$X \rightarrow Y$$

$$Y \rightarrow Z \mid b$$

$$Z \rightarrow bab$$

$$M \rightarrow ab$$

$$N \rightarrow a$$

Since $Z \rightarrow bab$, we add $Y \rightarrow bab \mid b$

$$S \rightarrow XY$$

$$X \rightarrow Y$$

$$Y \rightarrow bab \mid b$$

$$Z \rightarrow bab$$

$$M \rightarrow ab$$

$$N \rightarrow a$$

Since $Y \rightarrow bab \mid b$, we add $X \rightarrow bab \mid b$

$$S \rightarrow XY$$

$$X \rightarrow bab \mid b$$

$$Y \rightarrow bab \mid b$$

$$Z \rightarrow bab$$

$$M \rightarrow ab$$

$$N \rightarrow a$$

Since $Y \rightarrow bab \mid b$, we add $S \rightarrow Xbab \mid Xb$

$$S \rightarrow Xbab \mid Xb$$

$$X \rightarrow bab \mid b$$

$$Y \rightarrow bab \mid b$$

$$Z \to bab$$

$$M \to ab$$

$$N \to a$$

Since $X \to bab \mid b$, we add $S \to babbab \mid babb \mid bbab \mid bb$

$$S \to babbab \mid babb \mid bbab \mid bb$$

$$X \to bab \mid b$$

$$Y \to bab \mid b$$

$$Z \to bab$$

$$M \to ab$$

$$N \to a$$

Now remove the unreachable symbols, so production Rules will become

$$S \to babbab \mid babb \mid bbab \mid bb$$

Removal of NULL Productions

In a CFG, a Non-Terminal symbol A is a nullable variable if there is a production $A \to \Lambda$ or there is a derivation that starts at A and leads to Λ. Now, to remove NULL productions, following steps will be followed

Step 1: To remove $A \to \Lambda$, look for all productions whose right side contains A.

Step 2: Replace each occurrence of A in each of these productions with Λ.

Step 3: Add the resultant productions to the grammar.

Example 5

Remove NULL Productions from the grammar whose production rules are given by

$$S \to ABAC$$

$$A \to aA \mid \Lambda$$

$$B \to bB \mid \Lambda$$

$$C \to c$$

Solution:

Since there are two NULL productions, $A \to \Lambda$ and $B \to \Lambda$. So we remove both productions one by one. First to remove $A \to \Lambda$

$$S \to ABAC$$

$$S \to ABC \mid BAC \mid BC$$

$$A \to aA$$

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$$A \rightarrow a$$

Now new productions are

$$S \rightarrow ABAC \mid ABC \mid BAC \mid BC$$

$$A \rightarrow aA \mid a$$

$$B \rightarrow bB \mid \Lambda$$

$$C \rightarrow c$$

Now to remove $B \rightarrow \Lambda$

$$S \rightarrow ABAC \mid ABC \mid BAC \mid BC$$

$$S \rightarrow AAC \mid AC \mid C$$

$$B \rightarrow bB$$

$$B \rightarrow b$$

Now new productions are

$$S \rightarrow ABAC \mid ABC \mid BAC \mid BC \mid AAC \mid AC \mid C$$

$$A \rightarrow aA \mid a$$

$$B \rightarrow bB \mid b$$

$$C \rightarrow c$$

Now, our Productions have no NULL productions.

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Example 6

Remove NULL Productions from the grammar whose production rules are given by

$$S \rightarrow AB$$

$$A \rightarrow aAA \mid \Lambda$$

$$B \rightarrow bBB \mid \Lambda$$

Solution:

Since there are two NULL productions, $A \rightarrow \Lambda$ and $B \rightarrow \Lambda$. So, we remove both productions one by one. First to remove $A \rightarrow \Lambda$

$$S \rightarrow AB$$

$$S \rightarrow B$$

$$A \rightarrow aAA$$

$$A \rightarrow aA \mid a$$

Now new productions are

$$S \rightarrow AB \mid B$$

$$A \rightarrow aAA \mid aA \mid a$$

$$B \rightarrow bBB \mid \Lambda$$

Now to remove $B \rightarrow \Lambda$

$$S \rightarrow AB \mid B$$

$$S \rightarrow A \mid \Lambda$$

$$B \rightarrow bBB$$

$$B \rightarrow bB \mid b$$

Now combine all rules, new productions will be

$$S \rightarrow AB \mid B \mid A \mid \Lambda$$

$$A \rightarrow aAA \mid aA \mid a$$

$$B \rightarrow bBB \mid bB \mid b$$

Now non-terminal S also contains Λ because S itself is a nullable production.

Example 7:

Find a simplified grammar equivalent to the grammar G, having production rules

$$S \rightarrow AC \mid B$$

$$A \rightarrow a$$

$$C \rightarrow c \mid BC$$

$$E \rightarrow aA \mid e$$

Solution:

Here we follow all the steps to simplify the CFG i.e.

- Reduction of CFG.
- Removal of Unit Productions.
- Removal of NULL Productions.

Now first we reduce the CFG

Phase 1

$$\boldsymbol{T = \{a, c, e\}}$$

$$\boldsymbol{W_1 = \{A, C, E\}}$$

$$\boldsymbol{W_2 = \{A, C, E, S\}}$$

$W_3 = \{A, C, E, S\}$

$G' = \{(A, C, E, S), \{a, c, e\}, P, (S)\}$

P: $S \to AC$

$A \to a$

$C \to c$

$E \to aA \mid e$

Phase 2

$y_1 = \{S\}$

$y_2 = \{S, A, C\}$

$y_3 = \{S, A, C, a, c\}$

$y_4 = \{S, A, C, a, c\}$

$G'' = \{(A, C, S), \{a, c\}, P, (S)\}$

P: $S \to AC$

$A \to a$

$C \to c$

Now we remove the Unit productions from the CFG

As $A \to a$, we add $S \to aC$and remove $S \to AC$

P: $S \to aC$

$A \to a$

$C \to c$

As $C \to c$, we add $S \to ac$and remove $S \to aC$

P: $S \to ac$

$A \to a$

$C \to c$

Now remove the unreachable productions and simplified grammar will be

$$\boldsymbol{P}: \quad \boldsymbol{S \rightarrow ac}$$

Here, no NULL production is found. So simplified grammar is same as above.

-:ASSIGNMENT:-

Find the simplified CFG of the following given productions

Question No. 1

$$S \rightarrow aA \mid bB$$

$$A \rightarrow aS \mid bF$$

$$B \rightarrow aF \mid bS$$

$$F \rightarrow aB \mid bA \mid \Lambda$$

Question No. 2

$$S \rightarrow bS \mid aA$$

$$A \rightarrow bA \mid aF$$

$$F \rightarrow bF \mid \Lambda$$