

(Week16)Lecture31-32

Objectives: Learning objectives of this lecture are

- **RE to PDA Conversion**
 - **Examples**
- **FA to PDA Conversion (By CFG)**
 - **Examples**

Text Book & Resources: Introduction to Computer Theory – 2nd Edition – (I.O. Cohen)

Video Resources link

RE to PDA Conversion

PDA (Push Down Automata) supports all regular languages as well as Context Free Languages (CFLs). In our previous week, we have studied PDA for CFG's and CFL's. Now in this section, we will convert a regular language into its equivalent PDA by using the following steps.

- Regular Expression to CFG conversion
- CFG to PDA Conversion

Each regular expression can be converted into CFG and every CFG can be converted into PDA. We will explain this by the help of examples

Example: Write PDA for the given RE

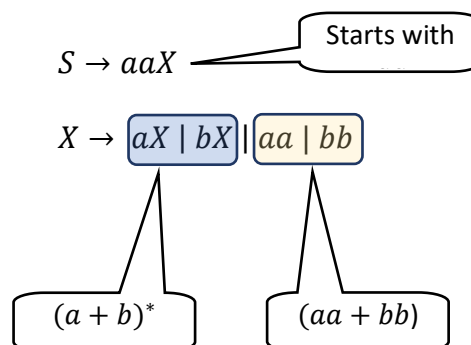
$$RE = aa(a + b)^*(aa + bb)$$

Solution: The Regular expression given is for the language starts with aa and end on double letter defined over $\Sigma = \{a, b\}$. Now first we convert this regular expression into CFG and then into its equivalent PDA. So, the CFG of above RE is

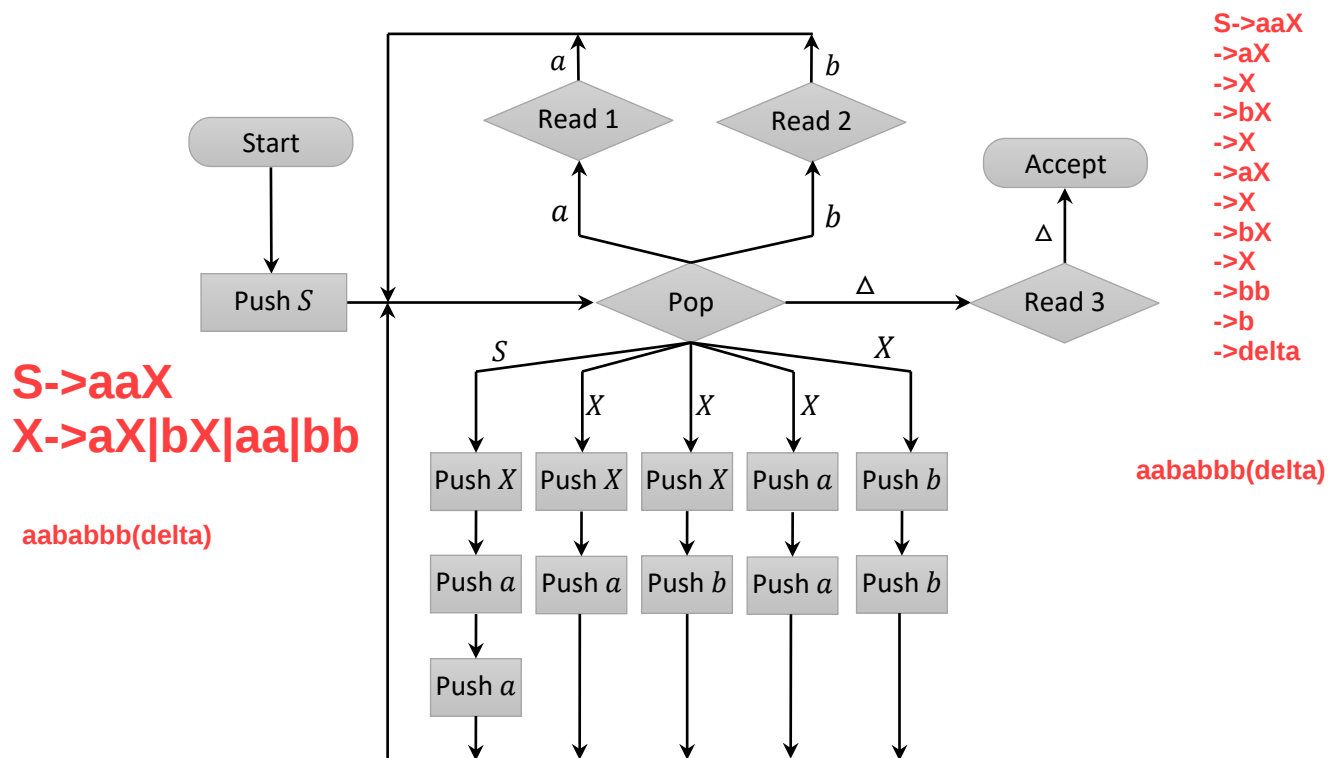
$$S \rightarrow aaX$$

$$X \rightarrow aX \mid bX \mid aa \mid bb$$

Here



Now we have to convert this CFG into PDA



Now we have to check the above PDA by passing a word. Let the word is *aababbbb*. Now this word created by the CFG as,

$$\begin{aligned}
 S &\Rightarrow aaX \\
 &\Rightarrow aabX \\
 &\Rightarrow aabaX \\
 &\Rightarrow aababX \\
 &\Rightarrow aababbb
 \end{aligned}$$

State	Stack	Tape
Start	$\Delta \dots$	<i>aababbbb</i> $\Delta\Delta \dots$
Push S	<i>S</i> $\Delta \dots$	<i>aababbbb</i> $\Delta\Delta \dots$
Pop	$\Delta \dots$	<i>aababbbb</i> $\Delta\Delta \dots$

Push X	$X \Delta \dots$	$aababbb \Delta \Delta \dots$
Push a	$aX \Delta \dots$	$aababbb \Delta \Delta \dots$
Push a	$aaX \Delta \dots$	$aababbb \Delta \Delta \dots$
Pop	$aX \Delta \dots$	$aababbb \Delta \Delta \dots$
Read 1	$aX \Delta \dots$	a $aababbb \Delta \Delta \dots$
Pop	$X \Delta \dots$	a $aababbb \Delta \Delta \dots$
Read 1	$X \Delta \dots$	aa $aababbb \Delta \Delta \dots$
Pop	$\Delta \dots$	aaa $aababbb \Delta \Delta \dots$
Push X	$X \Delta \dots$	aaa $aababbb \Delta \Delta \dots$
Push b	$bX \Delta \dots$	aaa $aababbb \Delta \Delta \dots$
Pop	$X \Delta \dots$	aaa $aababbb \Delta \Delta \dots$
Read 2	$X \Delta \dots$	$aaab$ $aababbb \Delta \Delta \dots$
Pop	$\Delta \dots$	$aaaba$ $aababbb \Delta \Delta \dots$
Push X	$X \Delta \dots$	$aaaba$ $aababbb \Delta \Delta \dots$
Push a	$aX \Delta \dots$	$aaaba$ $aababbb \Delta \Delta \dots$
Pop	$X \Delta \dots$	$aaaba$ $aababbb \Delta \Delta \dots$
Read 1	$X \Delta \dots$	$aaaba$ $aababbb \Delta \Delta \dots$
Pop	$\Delta \dots$	$aaabaa$ $aababbb \Delta \Delta \dots$
Push X	$X \Delta \dots$	$aaabaa$ $aababbb \Delta \Delta \dots$

Push b	$bX \Delta \dots$	$aababbb \Delta \Delta \dots$
Pop	$X \Delta \dots$	$aababbb \Delta \Delta \dots$
Read 2	$X \Delta \dots$	$aababbb \Delta \Delta \dots$
Pop	$\Delta \dots$	$aababbb \Delta \Delta \dots$
Push b	$b \Delta \dots$	$aababbb \Delta \Delta \dots$
Push b	$bb \Delta \dots$	$aababbb \Delta \Delta \dots$
Pop	$b \Delta \dots$	$aababbb \Delta \Delta \dots$
Read 2	$b \Delta \dots$	$aababbb \Delta \Delta \dots$
Pop	$\Delta \dots$	$aababbb \Delta \Delta \dots$
Read 2	$\Delta \dots$	$aababbb \Delta \Delta \dots$
Pop	$\Delta \dots$	$aababbb \Delta \Delta \dots$
Read 3	$\Delta \dots$	$aababbb \Delta \Delta \dots$
Accept	$\Delta \dots$	$aababbb \Delta \Delta \dots$

So, the above PDA passed the word belongs to the regular expression

Example: Write PDA for the given RE

$$RE = (ab + ba)(a + b)^*(ab + ba)$$

Solution: The Regular expression given is for the language cannot start and end with double letter defined over $\Sigma = \{a, b\}$.

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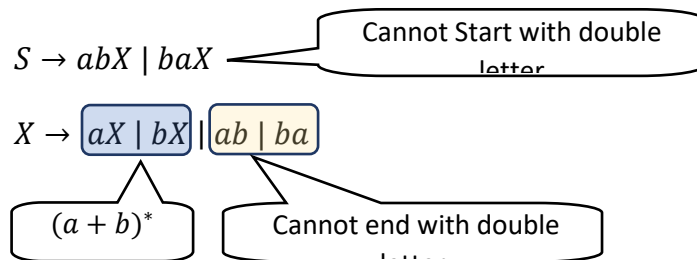
Now first we convert this regular expression into CFG and then into its equivalent PDA. So, the CFG of above RE is

$$S \rightarrow abX \mid baX$$

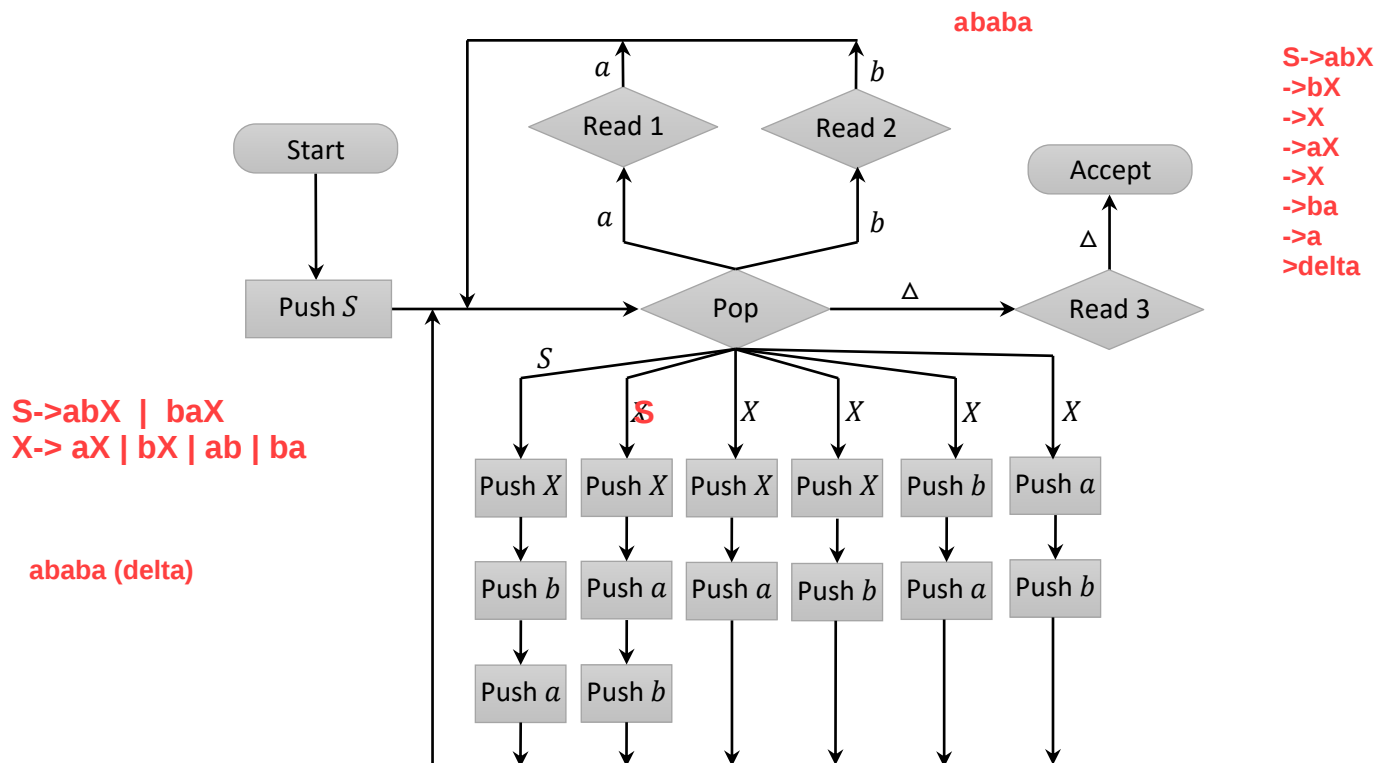
$$X \rightarrow aX \mid bX \mid ab \mid ba$$

$$R.E = (ab+ba)(a+b)^*(ab+ba)$$

Here



Now we have to convert this CFG into PDA



Now we have to check the above PDA by passing a word. Let the word is *ababa*. Now this word created by the CFG as,

$$S \Rightarrow abX$$

$$\Rightarrow abaX$$

$$\Rightarrow ababa$$

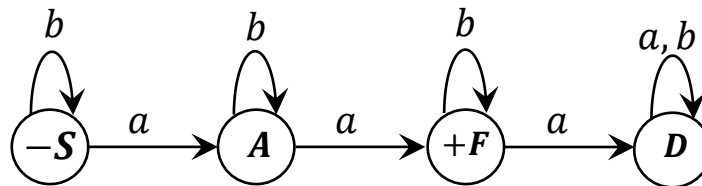
State	Stack	Tape
Start	$\Delta \dots$	<i>ababa</i> $\Delta\Delta \dots$
Push <i>S</i>	<i>S</i> $\Delta \dots$	<i>ababa</i> $\Delta\Delta \dots$
Pop	$\Delta \dots$	<i>ababa</i> $\Delta\Delta \dots$
Push <i>X</i>	<i>X</i> $\Delta \dots$	<i>ababa</i> $\Delta\Delta \dots$
Push <i>b</i>	<i>bX</i> $\Delta \dots$	<i>ababa</i> $\Delta\Delta \dots$

Push a	$abX \Delta \dots$	$ababa \Delta \Delta \dots$
Pop	$bX \Delta \dots$	$ababa \Delta \Delta \dots$
Read 1	$bX \Delta \dots$	$ababa$ $\Delta \Delta \dots$
Pop	$X \Delta \dots$	$ababa$ $\Delta \Delta \dots$
Read 2	$X \Delta \dots$	$ababa$ $\Delta \Delta \dots$
Pop	$\Delta \dots$	$ababa$ $\Delta \Delta \dots$
Push X	$X \Delta \dots$	$ababa$ $\Delta \Delta \dots$
Push a	$aX \Delta \dots$	$ababa$ $\Delta \Delta \dots$
Pop	$X \Delta \dots$	$ababa$ $\Delta \Delta \dots$
Read 1	$X \Delta \dots$	$ababa$ $\Delta \Delta \dots$
Pop	$\Delta \dots$	$ababa$ $\Delta \Delta \dots$
Push a	$a \Delta \dots$	$ababa$ $\Delta \Delta \dots$
Push b	$ba \Delta \dots$	$ababa$ $\Delta \Delta \dots$
Pop	$a \Delta \dots$	$ababa$ $\Delta \Delta \dots$
Read 2	$a \Delta \dots$	$ababa$ $\Delta \Delta \dots$
Pop	$\Delta \dots$	$ababa$ $\Delta \Delta \dots$
Read 1	$\Delta \dots$	$ababa$ $\Delta \Delta \dots$
Pop	$\Delta \dots$	$ababa$ $\Delta \Delta \dots$
Read 3	$\Delta \dots$	$ababa$ $\Delta \Delta \dots$

Accept	$\Delta \dots$	$ababa \Delta \Delta \dots$
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FSA to PDA Conversion (by CFG)

Example: Write PDA for the FSA given below



Solution:

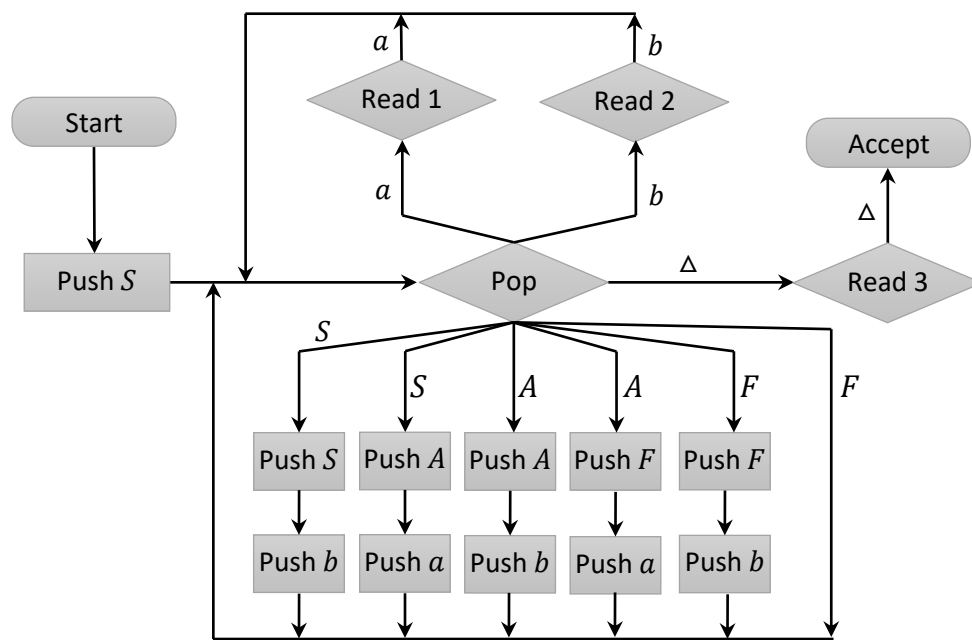
The above given FSA is of the language contain exactly two a 's defined over set of alphabets $\Sigma = \{a, b\}$. Now, to convert the given FSA into PDA, we will first convert this FSA into CFG, and then convert this CFG into PDA. So, the equivalent CFG of the above language will be,

$$S \rightarrow bS \mid aA$$

$$A \rightarrow bA \mid aF$$

$$F \rightarrow bF \mid \Lambda$$

Now, we will convert this CFG into PDA.



Now we have to check the above PDA by passing a word. Let the word, taken from FSA is *babab*. Then the table becomes like

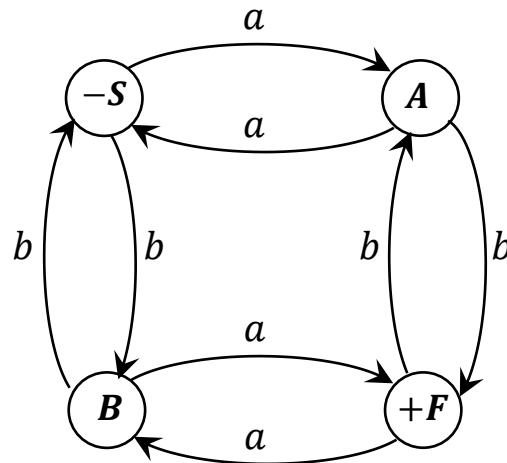
State	Stack	Tape
Start	$\Delta \dots$	<i>babab</i> $\Delta\Delta \dots$
Push <i>S</i>	<i>S</i> $\Delta \dots$	<i>babab</i> $\Delta\Delta \dots$
Pop	$\Delta \dots$	<i>babab</i> $\Delta\Delta \dots$
Push <i>S</i>	<i>S</i> $\Delta \dots$	<i>babab</i> $\Delta\Delta \dots$
Push <i>b</i>	<i>bS</i> $\Delta \dots$	<i>babab</i> $\Delta\Delta \dots$
Pop	<i>S</i> $\Delta \dots$	<i>babab</i> $\Delta\Delta \dots$
Read 2	<i>S</i> $\Delta \dots$	<i>babab</i> $\Delta\Delta \dots$
Pop	$\Delta \dots$	<i>babab</i> $\Delta\Delta \dots$

Push A	$A \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Push a	$aA \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Pop	$A \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Read 1	$A \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Pop	$\Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Push A	$A \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Push b	$bA \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Pop	$A \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Read 2	$A \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Pop	$\Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Push F	$F \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Push a	$aF \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Pop	$F \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Read 1	$F \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Pop	$\Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Push F	$F \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Push b	$bF \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Pop	$F \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$
Read 2	$F \Delta \dots$	$\cancel{b}abab \Delta \Delta \dots$

Pop	$\Delta \dots$	<i>babab</i> $\Delta \Delta \dots$
Pop	$\Delta \dots$	<i>babab</i> $\Delta \Delta \dots$
Read 3	$\Delta \dots$	<i>babab</i> $\Delta \Delta \dots$
Accept	$\Delta \dots$	<i>babab</i> $\Delta \Delta \dots$

So, the word passed successfully and our CFG is verified.

Example: Write PDA for the FSA given below



Solution:

The above given FSA is of the language contain odd number of a 's and odd number of b 's defined over set of alphabets $\Sigma = \{a, b\}$. Now, to convert the given FSA into PDA, we will first convert this FSA into CFG, and then convert this CFG into PDA. So, the equivalent CFG of the above language will be,

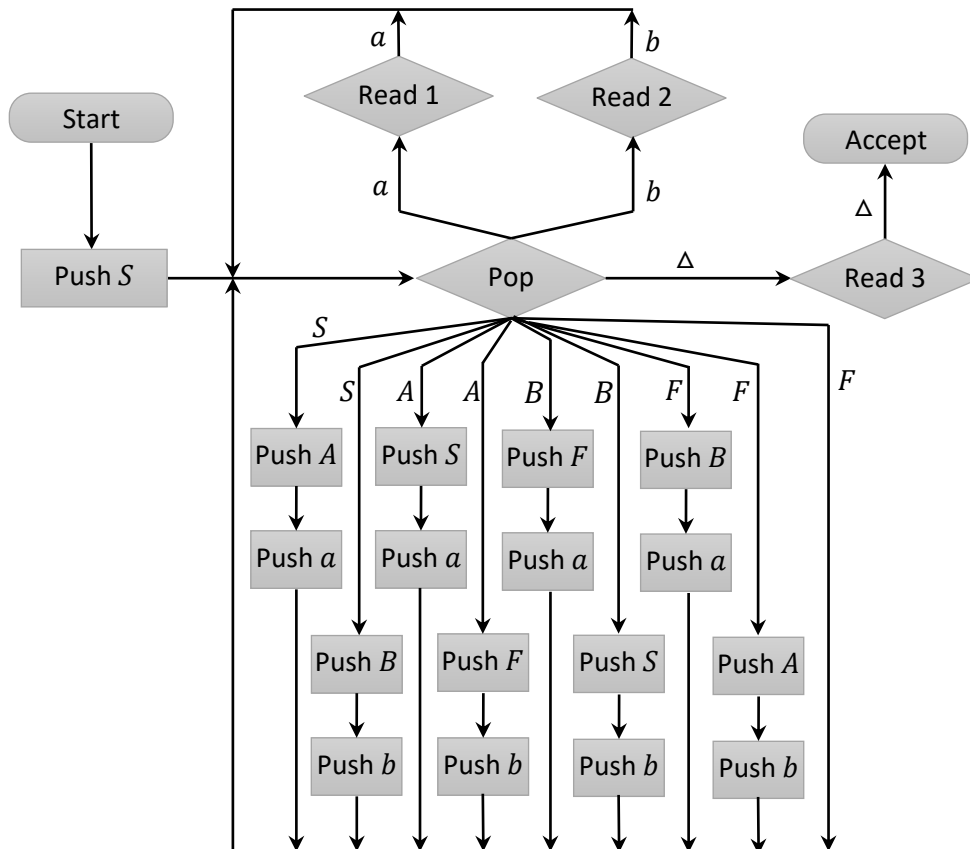
$$S \rightarrow aA \mid bB$$

$$A \rightarrow aS \mid bF$$

$$B \rightarrow aF \mid bS$$

$$F \rightarrow aB \mid bA \mid \Lambda$$

Now we will make the PDA of the above given CFG.



-:ASSIGNMENT:-

Question No. 1

Draw the PDA for the given regular expression.

$$RE = aa^*bab(ab)^*$$

Question No. 2

Draw the PDA for the following FSA by using CFG method.

